Review of Inference: Key Terms

1. **Estimation v. Inference**
   a. Estimation: What are your best parameter estimates? [BLUE]
   b. Inference: Are you sure/close? [confidence intervals; hypothesis tests]
2. **Confidence intervals**: Intervals generated in this fashion will contain the true parameter value some percentage of the time
3. **Hypothesis testing**: Can you reject the null hypothesis at some acceptable statistical significance level?

Our focus: Estimating the Population Mean

4. $Y \sim \mathrm{N}(\mu, \sigma^2)$ … random sampling: $\{Y_1, Y_2, \ldots Y_n\}$ iid ~ Y
5. Use $\overline{Y} = \frac{1}{n}\sum Y_i$ to estimate μ.
6. $\overline{Y} = \frac{1}{n}\sum Y_i \sim \mathrm{N}(\mu, \frac{\sigma^2}{n})$ … $\frac{\sigma}{\sqrt{n}}$ is the standard deviation (sd) of $\overline{Y}$ …
   $\frac{\overline{Y} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1)$
7. But the variance is unknown… so use the unbiased estimator $S_Y^2 = \frac{1}{n-1}\sum (Y_i - \overline{Y})^2$ to estimate σ^2
8. $\frac{S_Y}{\sqrt{n}}$ is the *Standard Error* (se) of the sample mean, and an estimator of $\frac{\sigma}{\sqrt{n}}$, sd($\overline{Y}$)
9. The **t statistic** is the **cornerstone of inference**: $\frac{\overline{Y} - \mu}{S_Y / \sqrt{n}} \sim t_{n-1}$
   a. Has a t distribution with n-1 dofs

Confidence Intervals

10. Confidence interval estimator: $\left[\overline{Y} - c\frac{S_Y}{\sqrt{n}},\ \overline{Y} + c\frac{S_Y}{\sqrt{n}}\right] = \left[\overline{Y} \pm c\frac{S_Y}{\sqrt{n}}\right]$
    a. c comes from the t statistic and t_{n-1}, the t distribution with n-1 dofs
    b. Effectively: estimate $\pm$ c std. errors (c=2 is not a bad place to start)

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Classical Hypothesis Testing

11. Null and Alternative Hypotheses

   a. H_0: the *Null* Hypothesis (the hypothesis we are testing)

   b. H_1: the *Alternative* Hypothesis, the alternative to H_0

   c. Almost always focus on H_0: The true parameter value is 0 $(\mu = 0)$

12. Two types of Error

   a. Type I – False Rejection; traditionally receives the most attention; *protecting the null*; reject the Null only if there is overwhelming evidence to the contrary

   b. Type II – False Acceptance

13. *Significance Level*: α

   a. the acceptable probability of a Type I error = P(Reject H_0 | H_0 is true)

   b. small is beautiful: 10%, 5%, 1% , …

14. Test: Reject the Null Hypothesis $H_0 : \mu = 0$ if $\left| \frac{\bar{Y}}{S_Y / \sqrt{n}} \right| > c_\alpha$

   a. c_α is defined by $P(|t_{n-1}| > c_\alpha) = \alpha$, the significance level (if t statistic is larger in magnitude than c_α)

   b. So: Reject the Null Hypothesis if the observed sample mean is at least c_α standard errors (*se's*) away from 0

15. *p value*: $P(|t_{n-1}| > \left| \frac{\bar{y}}{s_y / \sqrt{n}} \right|) = p$

   a. probability of being $\left| \frac{\bar{y}}{s_y / \sqrt{n}} \right|$ standard errors (*se's*) or further away from 0

16. Hypothesis testing is easier with p values: $p < \alpha \Leftrightarrow \left| \frac{\bar{y}}{s_y / \sqrt{n}} \right| > c_\alpha$

   a. Look for small p-values below the significance level

   b. You don't need to worry about different dofs (impacting critical values) …. just look at the p-value and see if it's small.